

Oscillation of spin polarization in a two-dimensional hole gas under a perpendicular magnetic field

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Abstract

Spin-charge coupling is studied for a strongly confined two-dimensional hole gas subject to a perpendicular magnetic field. The study is based on spin-charge coupled drift-diffusion equations derived from quantum-kinetic equations in an exact manner. The spin-orbit interaction induces an extra out-of-plane spin polarization. This contribution exhibits a persistent oscillatory pattern in the strong-coupling regime.

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INTRODUCTION

Recently, the study of spin-polarized transport in semiconductors has received much attention because of its potential applications in the field of semiconductor spintronics. Many authors have focused on spin-orbit interaction (SOI) that allows for purely electric manipulation of spin polarization in semiconductors. Beside this useful feature, SOI brings also into play the undesired spin relaxation due to the coupling between the momentum of charge carriers and their spin (c.f., for instance, Ref. [1]). Owing to this inhomogeneous broadening, each elastic and inelastic scattering mechanism opens up a spin dephasing channel [2]. The character of spin relaxation is quite different in systems with weak and strong SOI [3]. In the latter case, the magnetization can oscillate even in the absence of external fields. In contrast, for weakly spin-orbit coupled systems, the spin polarization decays exponentially unless it is permanently stimulated by external fields.

The decay of spin polarization seems to be unavoidable because of the non-conservation of the total spin. Nevertheless, a special persistent spin-precession pattern has been identified recently [4]. The infinite spin lifetime of this persistent spin helix occurs in a combined Rashba-Dresselhaus model at a certain wave vector that gives rise to a special spin rotation symmetry. Furthermore, oscillations of the nonequilibrium spin density in real space, which is induced by the Rashba SOI, have been reported in a number of recent papers [5, 6, 7]. These results on robust spin oscillations certainly encourage further experimental and theoretical studies of long-lived spin coherence states [8] in semiconductors with SOI.

In this paper, we focus on a strongly confined two-dimensional hole gas (2DHG) and study the mutual influence of SOI and a perpendicular external magnetic field. It is well known that a quantizing perpendicular magnetic field appreciably changes the transport properties of a two-dimensional electron gas (2DEG). The quantized energy spectrum manifests in Shubnikov-de Haas oscillations of the resistivity and may lead to the quantum-Hall effect. Due to the SOI-induced splitting and crossing of Landau levels, a beating pattern arises in Shubnikov-de Haas oscillations [9], which is used to determine the SOI strength from the measured magnetoresistivity. Similar quantum oscillations have been identified in the spin-relaxation rate [10]. Other studies [11, 12, 13] deal with the combined effects of Rashba and Dresselhaus SOI on the magnetotransport in a 2DEG. Unfortunately, comparable investigations of a 2DHG are limited although the SOI is much stronger in such systems.

We mention the analysis of transport equations for the 2DHG at zero magnetic field [14], the study of spin dephasing in p-type semiconductor quantum wells [15], and the treatment of the spin-Hall effect [16].

Our work is aimed to study the spin-charge coupled motion of holes in narrow quantum wells subject to a perpendicular magnetic field. Based on a rigorous density-matrix approach, spin-charge coupled drift-diffusion equations are derived for the 2DHG. In order to focus on general physical properties of the SOI in semiconductors, we adopt the simple cubic Rashba model that has been used in the literature [17, 18, 19, 20, 21] to simulate the SOI in a 2DHG. This model has the striking peculiarity that there is no coupling between the spin and charge components of the density matrix. One should contrast this finding with the linear Rashba model, which is used to study effects of SOI in a 2DEG. In this model, the SOI leads to a coupling between spin and charge degrees of freedom. For a 2DHG such a coupling is exclusively induced by external fields. Here, we treat a magnetic field applied perpendicular to the layer. Due to this field, the charge density and out-of-plane spin polarization couple to each other in the 2DHG. Consequently, an inhomogeneous spin polarization induces charge gradients, which are accompanied by an induced internal electric field calculated via Poisson's equation. The most interesting feature of our approach is, however, the observation that the character of the magnetic-field-induced spin-charge coupling differs qualitatively in the weak and strong coupling regime. For weak SOI, the dephasing time becomes much larger than the momentum-relaxation time so that the dominating mechanism is spin diffusion. In this regime, the field-induced magnetization exhibits only a smooth exponential dependence on spatial coordinates. Conversely, for strong SOI, the ballistic spin-transport regime is established, in which oscillations of the out-of-plane magnetization can occur. An experimental verification of this prediction would facilitate the technological exploitation of these long-lived spin states for the fabrication of logical gates.

BASIC THEORY

We treat coupled spin-charge excitations on the basis of an effective-mass Hamiltonian, which refers to the heavy-hole band of thin p-type quantum wells and which has been adopted in the literature [17, 18, 19, 20, 21] as an acceptable simple approximation. Our

model includes short-range spin-independent elastic scattering on impurities and a constant perpendicular magnetic field B , from which only the Zeeman splitting is considered. The related heavy-hole Hamiltonian of the cubic Rashba model has the second-quantized form

$$H = \sum_{\mathbf{k}, \lambda} a_{\mathbf{k}\lambda}^\dagger [\varepsilon_{\mathbf{k}} - \varepsilon_F] a_{\mathbf{k}\lambda} - \sum_{\mathbf{k}, \lambda, \lambda'} (\hbar \vec{\omega}_{\mathbf{k}} \cdot \vec{\sigma}_{\lambda\lambda'}) a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda'} + u \sum_{\mathbf{k}, \mathbf{k}'} \sum_{\lambda} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}'\lambda}, \quad (1)$$

where $a_{\mathbf{k}\lambda}^\dagger$ ($a_{\mathbf{k}\lambda}$) denote the creation (annihilation) operators with in-plane quasi-momentum $\mathbf{k} = (k_x, k_y, 0)$ and spin λ . In Eq. (1), we introduced the Fermi energy ε_F , the vector of Pauli matrices $\vec{\sigma}$, and the strength u of the 'white-noise' elastic impurity scattering, which gives rise to the momentum relaxation time τ . The heavy-hole band is described by the dispersion relation $\varepsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / (2m)$. The coupling of spin states as described by

$$\hbar \vec{\omega}_{\mathbf{k}} = \left[i \frac{\alpha}{2} (k_+^3 - k_-^3), \frac{\alpha}{2} (k_+^3 + k_-^3), \hbar \omega_c \right], \quad (2)$$

is due to the Zeeman splitting $\hbar \omega_c = g^* \mu_B B / 2$ and the SOI, the strength of which is denoted by α . In Eq. (2), we have $k_{\pm} = k_x \pm i k_y$, $k_x = k \cos(\varphi)$, $k_y = k \sin(\varphi)$, and $\hbar \omega_k = \alpha k^3$. Within the Born approximation with respect to elastic impurity scattering, the four components $(f, \vec{f}) = (\sum_{\lambda} f_{\lambda}^{\lambda}, \sum_{\lambda, \lambda'} f_{\lambda}^{\lambda'} \vec{\sigma}_{\lambda\lambda'})$ of the spin-density matrix $f_{\lambda}^{\lambda'}$ satisfy the following Laplace-transformed quantum-kinetic equations [22, 23]

$$s f - \frac{i\hbar}{m} (\boldsymbol{\kappa} \cdot \mathbf{k}) f + i \vec{\omega}_{\boldsymbol{\kappa}}(\mathbf{k}) \cdot \vec{f} = \frac{1}{\tau} (\bar{f} - f) + f_0, \quad (3)$$

$$\begin{aligned} & s \vec{f} + 2(\vec{\omega}_{\mathbf{k}} \times \vec{f}) - \frac{i\hbar}{m} (\boldsymbol{\kappa} \cdot \mathbf{k}) \vec{f} + i \vec{\omega}_{\boldsymbol{\kappa}}(\mathbf{k}) f \\ &= \frac{1}{\tau} (\bar{\vec{f}} - \vec{f}) + \frac{1}{\tau} \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \overline{f \hbar \vec{\omega}_{\mathbf{k}}} - \frac{\hbar \vec{\omega}_{\mathbf{k}}}{\tau} \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \bar{f} + \vec{f}_0, \end{aligned} \quad (4)$$

in which the SOI-dependent vector

$$\hbar \vec{\omega}_{\boldsymbol{\kappa}}(\mathbf{k}) = 3\alpha [(k_y^2 - k_x^2) \kappa_y - 2k_x k_y \kappa_x, (k_x^2 - k_y^2) \kappa_x - 2k_x k_y \kappa_y, 0] \quad (5)$$

couple the spin and charge degrees of freedom to each other. The wave vector $\boldsymbol{\kappa}$ refers to the center-of-mass motion and disappears in models that refer to homogeneous spin and charge distributions. Initial charge and spin densities are denoted by $f_0 = n$ and \vec{f}_0 , respectively. The cross line over \mathbf{k} -dependent functions indicates an integration over the polar angle φ of the in-plane vector \mathbf{k} . s denotes the variable of the Laplace transformation and takes over the role of the time parameter t .

By treating the kinetic Eqs. (3) and (4) in the long-wavelength limit, coupled spin-charge drift-diffusion equations are derived for the angle-averaged spin-density matrix $(\bar{f}, \bar{\mathbf{f}})$. The method has already been applied to a 2DEG without any external fields [23]. In this approach, it is assumed that carriers quickly reestablish thermal equilibrium. This fact justifies the ansatz $\bar{f}(\varepsilon_{\mathbf{k}}, \boldsymbol{\kappa} | s) = n(\varepsilon_{\mathbf{k}})F(\boldsymbol{\kappa} | s)$, where $n(\varepsilon_{\mathbf{k}})$ denotes the Fermi distribution function. Expanding the solution of Eqs. (3) and (4) up to second order in $\boldsymbol{\kappa}$ and calculating the integral over the angle φ , we obtain our main theoretical result namely the following spin-charge coupled drift-diffusion equations

$$(s + D_0\kappa^2)\bar{f} - \Gamma_z\kappa^2\bar{f}_z = n, \quad (6)$$

$$(s + \frac{1}{\tau_{sz}} + D_z\kappa^2)\bar{f}_z + \Gamma_0\bar{f} = f_{z0}, \quad (7)$$

$$(\sigma_0^2 s\tau + 2\Omega^2(2s\tau + 1))\bar{f}_x + D_x\tau\kappa^2\bar{f}_x - 2\sigma_0\omega_c\tau(1 + \tilde{D}\tau\kappa^2)\bar{f}_y = (\sigma_0^2 + 2\Omega^2)\tau f_{x,0}, \quad (8)$$

$$(\sigma_0^2 s\tau + 2\Omega^2(2s\tau + 1))\bar{f}_y + D_x\tau\kappa^2\bar{f}_y + 2\sigma_0\omega_c\tau(1 + \tilde{D}\tau\kappa^2)\bar{f}_x = (\sigma_0^2 + 2\Omega^2)\tau f_{y,0}, \quad (9)$$

where we used the abbreviations $\sigma_0 = s\tau + 1$ and $\Omega = \omega_c\tau$. The k -dependent coefficients in this set of equations have the form

$$D_0 = \frac{D}{\sigma_0^2}, \quad \Gamma_z = 24\frac{\hbar\omega_c\tau}{m}\Omega^2\frac{\sigma_0^2 + 2\Omega^2}{\sigma_0^2(\sigma_0^2 + 4\Omega^2)^2}, \quad (10)$$

$$\frac{1}{\tau_{sz}} = \frac{4\Omega^2}{\sigma_0\tau}, \quad D_z = D\frac{\sigma_0^2 - 12\Omega^2}{(\sigma_0^2 + 4\Omega^2)^2}, \quad \Gamma_0 = -\frac{\chi H}{\sigma_0\mu_B\tau_{sz}}, \quad (11)$$

$$D_x = D\frac{\sigma_0^6 + 24\sigma_0^2\Omega^4 + 32\Omega^6}{\sigma_0^2(\sigma_0^2 + 4\Omega^2)^2}, \quad \tilde{D} = D\frac{4\Omega^2 - 3\sigma_0^2}{(\sigma_0^2 + 4\Omega^2)^2}, \quad (12)$$

where we introduced the diffusion coefficient $D = v^2\tau/2$, the Bohr magneton μ_B , and magnetic susceptibility χ . The Eqs. (6) to (9) completely decouple in the absence of the external magnetic field, when $\Gamma_z = \Gamma_0 = \omega_c = 0$. This is a peculiarity of the cubic Rashba model. When a perpendicular magnetic field is applied to the 2DHG, a steady-state out-of-plane spin polarization arises

$$f_z^{(0)} = -\hbar\omega_c n' = \frac{\chi H}{\mu_B}, \quad (13)$$

which couples to the charge density. For a 2DEG the situation is different. In this case, the out-of-plane spin polarization couples to the in-plane spin components [23]. The most surprising feature of our solution exhibits the spin-diffusion coefficient D_z in Eq. (11), the form of which agrees with a recently published result [14, 23] derived by an alternative approach. This particular diffusion coefficient becomes negative for strong SOI ($\Omega > \sigma_0/\sqrt{12}$)

indicating an instability of the spin system. In this regime, spin diffusion has the tendency to strengthen initial spin fluctuations. The competition between this self-strengthening and spin relaxation leads to undamped spin oscillations that are characteristic for ballistic spin transport. Such spin oscillations result from the coupling between the charge density and the out-of-plane spin polarization expressed by Eqs. (6) and (7). What is interesting is that this unusual result for D_z can only be obtained by taking into account the off-diagonal elements of the density matrix. (In fact, neglecting f_x and f_y in Eq. (4), we obtain simply $D_z = D$). Therefore, the oscillations in the strong SOI regime have a pure quantum-mechanical origin that is manifested in the quasi-classical Eqs. (6) and (7). Strictly speaking, this result arises beyond the applicability of the drift-diffusion approach [23, 24].

The time dependence of the in-plane spin polarization as described by Eqs. (8) and (9) is governed by characteristic poles [22, 25] that are calculated from $\sigma_0^2 s\tau + 2\Omega^2(2s\tau + 1) = 0$. Let us treat the strong-coupling regime $\Omega \gg 1$ for the in-plane spin polarization that is determined by poles at $s\tau = -3/4 \pm 2i\Omega$. Performing the inverse Laplace and Fourier transformations, we obtain for the spectral spin polarization the result

$$\bar{f}_x(\mathbf{r}, k | t) = \exp \left[-\frac{\mathbf{r}^2}{16Dt} - \frac{3t}{4\tau} \right] \left\{ \frac{\cos(2\omega_k t)}{t/\tau} f_{x0} - \frac{\omega_c \tau}{2\Omega} \sin(2\omega_k t) f_{y0} \right\} / (32\pi D), \quad (14)$$

$$\bar{f}_y(\mathbf{r}, k | t) = \exp \left[-\frac{\mathbf{r}^2}{16Dt} - \frac{3t}{4\tau} \right] \left\{ \frac{\cos(2\omega_k t)}{t/\tau} f_{y0} + \frac{\omega_c \tau}{2\Omega} \sin(2\omega_k t) f_{x0} \right\} / (32\pi D), \quad (15)$$

which describes damped oscillations of an initially at $\mathbf{r} = \mathbf{0}$ injected spin packet. The external magnetic field couples initial nonvanishing in-plane spin components to each other. A spot like initial in-plane spin polarization could be produced in experiment by a short laser pulse. The evolution of this initial inhomogeneous spin distribution is described by Eqs. (14) and (15).

SPIN POLARIZATION FOR A STRIPE GEOMETRY

In this Section, the magnetic-field-induced coupling between the charge distribution \bar{f} and the out-of-plane spin polarization \bar{f}_z in a 2DHG is treated in more detail for a stripe of width $2L$ oriented along the x axis. To this end, the steady-state solution ($s = 0$, $\sigma_0 = 1$) of Eqs. (6) and (7) is transformed back to the representation in spatial coordinates x and y . Due to the considered stripe geometry, the densities are independent of x . The variation of the charge density $\bar{f}(k, y)$ induces a self-consistent internal electric field $E_y(k, y)$ that is

calculated from the Poisson equation. This internal in-plane electric field is a by-product of the spin-charge coupling. Its reaction on the spin is accounted for by drift terms in Eqs. (6) and (7). Its phenomenological consideration in Eq. (6) for the carrier density is ruled by the concept of effective chemical potential [22]. Motivated by studies of electric-field effects on spin transport, we introduce a similar contribution in Eq. (7) for the out-of-plane spin polarization. Putting all together, the following set of coupled equations for spin-charge excitations are obtained

$$D(k)\bar{f}'(k, y) - \mu E_y(k, y)\bar{f}(k, y) - \Gamma_z(k)\bar{f}_z'(k, y) = 0, \quad (16)$$

$$D_z(k)\bar{f}_z''(k, y) - \mu E_y(k, y)\bar{f}_z'(k, y) - \frac{1}{\tau_{sz}(k)}\bar{f}_z(k, y) - \Gamma_0(k)\bar{f}(k, y) = 0, \quad (17)$$

$$E_y'(k, y) = \frac{4\pi e}{\varepsilon}(\bar{f}(k, y) - n(k)), \quad (18)$$

in which $\mu = e\tau/m$ denotes the mobility and ε is the dielectric constant. Primes indicate derivatives with respect to y . We derive an analytical solution of these equations by calculating the lowest-order contributions in the induced electric field E_y . Within this perturbational schema, we make the ansatz $\bar{f} = n + \Delta\bar{f}$ and $\bar{f}_z = f_z^{(0)} + \Delta\bar{f}_z$, where the corrections result from the spin-charge coupling $\Delta\bar{f}$, $\Delta\bar{f}_z \sim E_y$. In addition, hard-wall boundary conditions $\Delta\bar{f}_z(\pm L) = 0$ and the existence of interface charges $E_y(\pm L) = \pm E_0$ are assumed. We obtain the analytic solution

$$\Delta\bar{f}_z = \frac{\lambda_1\lambda_2 E_0\Gamma_0}{4\pi e/\varepsilon} \frac{\cosh(\lambda_2 L) \cosh(\lambda_1 y) - \cosh(\lambda_1 L) \cosh(\lambda_2 y)}{N(L)}, \quad \frac{E_y}{E_0} = \frac{N(y)}{N(L)}, \quad (19)$$

where $\lambda_{1,2}$ are calculated from the secular equation

$$\left(D\lambda^2 - \frac{4\pi e}{\varepsilon}\mu n\right) \left(D_z\lambda^2 - \frac{1}{\tau_{sz}}\right) - \lambda^2\Gamma_0\Gamma_z = 0. \quad (20)$$

In Eq. (19), the abbreviation

$$N(y) = \lambda_2 \left(D_z\lambda_1^2 - \frac{1}{\tau_{sz}}\right) \cosh(\lambda_2 L) \sinh(\lambda_1 y) - \lambda_1 \left(D_z\lambda_2^2 - \frac{1}{\tau_{sz}}\right) \cosh(\lambda_1 L) \sinh(\lambda_2 y) \quad (21)$$

was introduced. For weak magnetic fields $\omega_c\tau \ll 1$, we obtain the final result

$$\Delta f_z(\varepsilon_{\mathbf{k}}, y) = -\frac{eE_0\chi H\lambda_1^2 L_D^3}{\mu_B(1 - (\lambda_1 L_D)^2)} \coth(L/L_D) \left\{ \frac{\cosh(y/L_D)}{\cosh(L/L_D)} - \frac{\cosh(\lambda_1 y)}{\cosh(\lambda_1 L)} \right\} \frac{dn(\varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}}}, \quad (22)$$

with

$$\lambda_1 = 1/\sqrt{D_z\tau_{sz}}, \quad \lambda_2 = \sqrt{4\pi e\mu n/(D\varepsilon)} = L_D^{-1}, \quad (23)$$

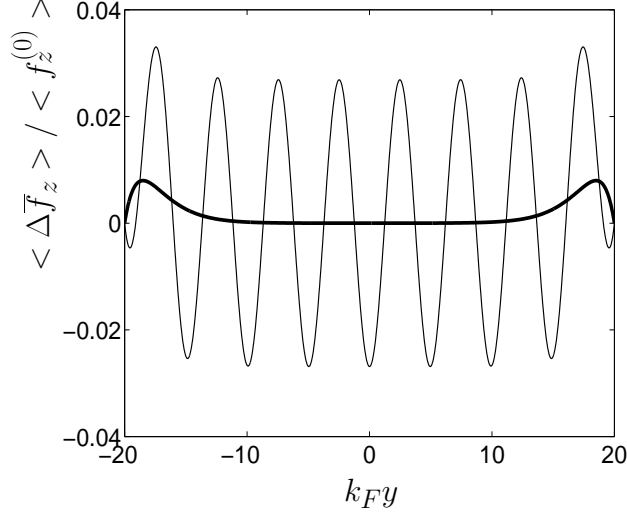


FIG. 1: SOI-induced out-of-plane spin polarization obtained from Eq. (22) by integrating over $\varepsilon_{\mathbf{k}}$ (indicated by $\langle \dots \rangle$). At zero temperature, all quantities are calculated at the Fermi momentum k_F . Parameters used in the calculation are: $k_F = 0.1 \text{ nm}^{-1}$, $m\alpha/\hbar^2 = 2 \text{ nm}$, $E_0 = 100 \text{ V/cm}$, and $n = 10^{15}/\text{cm}^3$. The thick and thin lines refer to weak ($\Omega = 0.173$, $\tau = 0.05 \text{ ps}$) and strong ($\Omega = 0.34$, $\tau = 0.1 \text{ ps}$) spin-orbit coupling, respectively.

where L_D denotes the Debye screening length. Again, we meet a peculiarity of the cubic Rashba model for a 2DHG. The final integral over the energy $\varepsilon_{\mathbf{k}}$ is easily calculated at low temperatures. Due to the factor $dn(\varepsilon_{\mathbf{k}})/d\varepsilon_{\mathbf{k}}$, the field-induced spin polarization is exclusively determined by energies at the Fermi surface for a degenerate hole gas. Therefore, the recently studied inhomogeneous broadening [15] due to elastic scattering is ineffective in this regime. However, inelastic scattering, which we disregarded in this work, may play an essential role for the formation of a persistent oscillatory spin pattern at strong SOI.

The character of the solution for the out-of-plane spin polarization mainly depends on the strength of the SOI. In weakly coupled systems ($\Omega < 1/\sqrt{12}$, $D_z > 0$), the spin polarization exhibits an exponential dependence as shown by the thick line in Fig. 1. The self-consistent coupling between spin and charge degrees of freedom leads to an excess magnetization at the boundaries of the stripe. The picture changes dramatically, when we consider the strong-coupling regime ($\Omega > 1/\sqrt{12}$, $D_z < 0$). In this case, the wave number λ_1 becomes imaginary giving rise to spin-coherent oscillations. An example for this persistent spin pattern is shown by the thin line in Fig. 1. Despite the included elastic scattering on impurities,

the spin lifetime of these oscillations is infinite in the strong-coupling regime. Moreover, the oscillation amplitude is considerably enhanced at the resonance $\lambda_1 L = (2n + 1)\pi/2$ with n being any integer. A similar enhancement has been predicted for the SOI-induced zitterbewegung [26]. This observation also remind of a Fabry-Perot interferometer in optics. Like the finesse of the interferometer diverges for perfect reflective mirrors, the amplitude of the spin oscillations becomes infinite for the above mentioned particular values of the spin-orbit coupling and the width of the stripe. This idealized behavior indicates that beside elastic scattering on impurities also other spin-relaxation mechanisms have to be taken into account for a more realistic description of spin excitations at strong SOI. The experimental observation of the interesting persistent oscillatory spin structure is certainly challenging. It requires a spin detection set up with a high spatial resolution (the typical wavelength of the oscillations is of the order of 100 nm). As the magnetic field leads to a coupling between the out-of-plane spin polarization and the charge density, both the induced internal electric field and the charge density exhibit similar oscillations in the strong coupling regime.

SUMMARY

We studied a 2DHG with SOI and elastic impurity scattering under the influence of a perpendicular magnetic field. Applying an exact procedure, spin-charge coupled drift-diffusion equations were derived from quantum-kinetic equations for the spin-density matrix. The magnetic field mainly causes a coupling between the out-of-plane spin polarization and the charge density. The character of effects that result from this coupling strongly depend on the strength of the SOI. For weak SOI ($\Omega < 1/\sqrt{12}$), spin diffusion gives rise to an exponential decay of an initial spin polarization. In contrast, for strong SOI, the spin transport exhibits ballistic character so that oscillations of the magnetization can occur. This general conclusion was illustrated by a treatment of the spin polarization in a stripe composed of a 2DHG. The magnetic field induces a background magnetization that is superimposed by a contribution stemming from the SOI. The excess magnetization, which results from the spin-charge coupling, exhibits a persistent oscillatory spin pattern for systems with strong spin-orbit coupling. Similar standing and propagating spin oscillations with wavelength down to several nanometers have been treated for thin magnetic film samples [27]. The application of this mechanism for spin-wave logic gates depends on whether short-wavelength

spin oscillations can be manipulated and detected by a suitable experimental set up.

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